On combining the Bernoulli and Poiseuille equation—A plea to authors of college physics texts

Costas Emmanuel Synolakis
School of Engineering, University of Southern California, Los Angeles, California 90089-0242

Henry S. Badeer
Department of Physiology, Creighton University School of Medicine, Omaha, Nebraska 68178-0224

(Received 28 April 1988; accepted for publication 5 December 1988)

A generalized form of the Bernoulli equation is presented. The assumptions involved in the derivation and its limitations are identified. The usual form of the Bernoulli and Poiseuille equations is shown to be a special case of this generalized equation. Various implications in the teaching of college physics are discussed.

The Bernoulli equation[1],

$$\tfrac{1}{2}V^2 + p/\rho + gz = \text{const}, \qquad (1)$$

where p is the pressure, ρ is the density, V is the magnitude of the velocity vector, g is the acceleration of gravity, and z is the elevation, is undoubtedly one of the most commonly used equations in physics and it is often referred to as the most fundamental relationship of fluid mechanics.

Even though this equation is described in most introductory college physics texts, the assumptions implicit in (1) and its limitations are not. In the context of viscous flow, most texts discuss another equation, the Poiseuille equation, separately, creating the unfortunate impression that Bernoulli's equation is not applicable for viscous flow.

As teachers of undergraduate and graduate courses in physiology and geophysical fluid dynamics, we have noted a large number of students with the preconceived notion that the Bernoulli and Poiseuille equations are mutually exclusive. This notion seems to be so strong as to survive occasionally even introductory undergraduate fluid mechanics classes. We have also noted confusion in the application of Bernoulli's equation; many students understand that it is always applicable in inviscid flow, others think that it can only be used for steady flow, and still others are uncertain whether the gravity term is included. Very frequently, students are confused as to whether Bernoulli's equation represents conservation of energy or momentum or whether it is applicable for "streamline flow" or for turbulent flow.

On close scrutiny of many introductory physics and even some fluid mechanics texts, we have found that this confusion is a reflection of rather brief and often wrong discussions of the limitations of the Bernoulli and Poiseuille equations. This is unnecessary because the algebra involved in the derviation of these equations is straightforward.

In the spirit of improving the accuracy and effectiveness of physics texts, we will present here a generalized form of Bernoulli's equation appropriate for college students, and we shall show that the usual forms of the Bernoulli and Poiseuille equations can be derived as special cases of this equation. We will carefully outline the limitations of these equations and will provide a comprehensive discussion of