The Bernoulli-Poiseuille Equation

By Henry S. Badeer and Costas E. Synolakis

College physics texts present the Bernoulli equation (which describes the flow of inviscid fluids) as the most useful equation in fluid dynamics, emphasizing the restrictions that apply. Some texts also discuss the Poiseuille equation, which deals only with viscous flow, excluding gravitational and accelerative forces. We suggest that a combination of the two equations is desirable for a more general application.

Our approach is conceptual, uses helpful illustrations, and is aimed at undergraduate students taking the college physics course in preparation for biomedical or engineering sciences. We will use the following notations:

P	=	fluid pressure
ΔV or Q	=	volume of a fluid element passing through a cross-sectional area per unit time
$P\Delta V$	=	pressure-volume work
$\bar{v}$	=	average speed over a cross section
A	=	cross-sectional area
R	=	tube radius
ρ	=	density
η	=	viscosity
g	=	gravitational acceleration
h	=	vertical distance between two points in flow tube

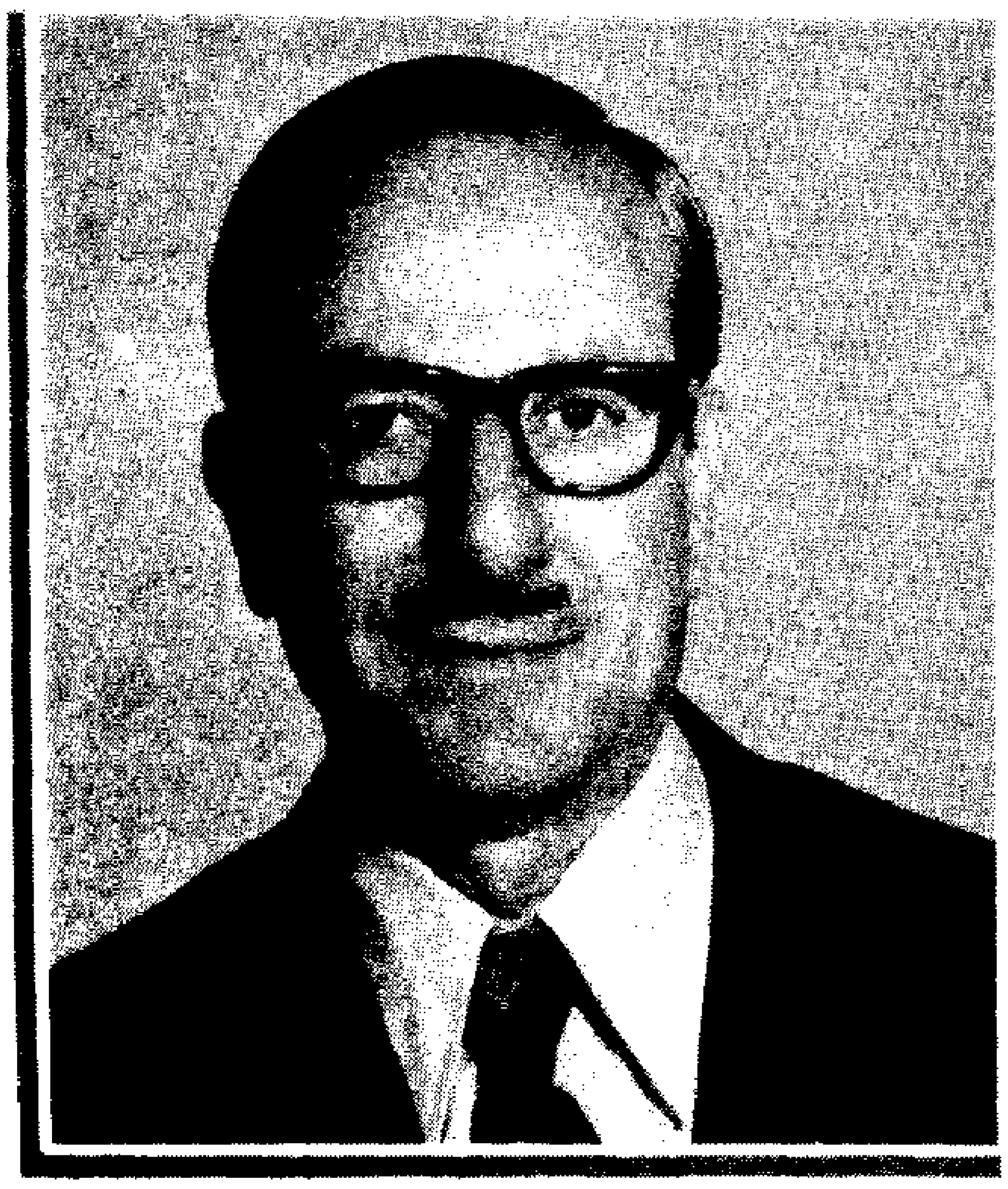
The standard form of Bernoulli's equation[1]

$$P/\rho + gh + \bar{v}^2/2 = \text{a constant} \qquad (1)$$

is valid between any two points in incompressible, steady, irrotational, inviscid flow. Under these conditions, the average velocity $\bar{v}$ over a cross section is used often instead of local velocity v. When the flow is rotational (i.e., vorticity exists), the equation is valid along a streamline only. The equation also can be extended to include viscous effects.[1]

Equation (1) expresses conservation of energy (E) and it has three terms or components when describing nonviscous flow. These components are a) the energy related to do work against the pressure of the liquid, b) the energy related to the elevation of the liquid (i.e., the gravitational potential energy), and c) the energy related to changes in the speed of the liquid or kinetic energy (KE). Any combination of these components may be absent in a given situation.

Figure 1, often used in physics texts, illustrates these three components. A frictionless liquid is driven through a rigid tube, of varying cross-sectional area, that is oriented vertically. Work is done against the force resulting from the pressure acting on the fluid element, and also work is expended to increase its gravitational potential energy. Since the flow tube narrows progressively, there is an increase in the speed of the liquid as a consequence of the continuity equation ($A_1\bar{v}_1 = A_3\bar{v}_3$), resulting in a change in its kinetic energy. To facilitate the understand-



Henry S. Badeer *received his M.D. degree from the American University of Beirut, Lebanon, and taught medical physiology there for many years. He emigrated to Brooklyn, New York, where he was a visiting professor at Downstate Medical Center. Since 1967, he has been professor of physiology at Creighton University School of Medicine. His specialty is the physiology and biophysics of circulation, and he has published a textbook for medical students.*