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ROOTS OF $J_\gamma(z) \pm iJ_{\gamma+1}(z) = 0$ AND THE EVALUATION OF INTEGRALS WITH CYLINDRICAL FUNCTION KERNELS

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Abstract. An elementary proof is presented showing that the function $f(z) = J_\gamma(z) \pm iJ_{\gamma+1}(z)$, where γ is a natural number, has no zeroes in the lower and upper half-planes respectively. The roots of $f(z)$ are given for certain values of γ and their locations are plotted. Cartesian maps (mappings of constant coordinate lines) of $f(z)$ are obtained, and special features of these maps are discussed. Some integrals with cylindrical kernels involving $f(z)$ are obtained in terms of the zeroes of $J_\gamma(z)$.

I. Introduction. The function

$$J_0(z) - iJ_1(z) = 0$$

frequently arises in certain solutions of problems in coastal hydrodynamics [1]. Its behaviour in the upper half-plane has been well established by Synolakis [1] and Rawlins [2], while its zeroes in the lower half-plane were first calculated by Macdonald [3]. In this paper we will present a theorem discussing the behaviour of the function

$$f(z) = J_\gamma(z) \pm iJ_{\gamma+1}(z) = 0 \tag{1}$$

and certain applications. This function arises in problems of wave reflection off composite beaches, i.e., beaches with multiple slopes.

II. THEOREM. The function $f(z) = J_\gamma(z) \pm iJ_{\gamma+1}(z)\ \forall\gamma\varepsilon N$, where N is a natural number has no zeroes in the lower and upper half-planes respectively.

Proof. Let the sequence $\zeta_{\gamma,n}$, where γ is a natural number, denote the real zeroes of $J_\gamma(z)$ which lie on the positive real axis arranged in order of nondecreasing magnitude other than the origin. Using the result of Bateman [4]

$$\frac{J_{\gamma+1}(z)}{J_\gamma(z)} = -2z\sum_{n=1}^{\infty}\frac{1}{(z^2 - \zeta^2_{\gamma,n})}, \tag{2}$$

and using the fact that $J_\gamma(z)$ and $J_{\gamma+1}(z)$ have no common zeroes other than the origin [5], and generalizing the method of Rawlins [2], one can derive that the zeroes

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